Pinner Park Junior School

Mathematics Calculation Policy

2015-2016

Aims and principles

Our aim at PPJS is that when children leave primary school they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers.
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally.

Mental methods of calculation (fluency in number)

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills.

Secure mental calculation requires the ability to:

- recall key number facts instantly – for example:

  all addition and subtraction facts for each number to at least 10 (Year 2)

  sums and differences of multiples of 10 (Year 3)

  multiplication facts up to 12 x 12 (Year 4)

- use taught strategies to work out the calculation – for example:

  recognise that addition can be done in any order and use this to add mentally a one- digit number or a multiple of 10 to a one-digit or two-digit number

  partition two-digit numbers in different ways including multiples of ten and one; add the tens and ones separately and then recombine.

- understand how the rules and laws of arithmetic are used and applied – for example:

  - to add or subtract mentally combinations of one-digit and two-digit numbers (Year 3)

  - to calculate mentally with whole numbers and decimals (Year 6).

The revised 2014 curriculum contains the requirements for each year group regarding mental strategies. The 1999 NNS Framework contains extensive examples of mental calculation strategies that need to be taught and assessed regularly throughout. The new curriculum has shifted various objectives from one year group to another so care must be taken when using the examples and guidance of the previous framework.

MENTAL CALCULATION STRATEGIES ARE TO BE USED CONTINUOUSLY. THEY ARE NOT REPLACED BY WRITTEN METHODS.

Written Methods of Calculation

The 1999 Framework sets out progression in written methods of calculation that highlights how children would move from informal methods of recording to expanded methods that are staging posts to a compact written method for each of the four operations.

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding.

This guidance promotes the use of what are commonly known as 'standard' written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator.

We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that we adopt a consistent whole school approach to calculation that all teachers understand and towards which they work. We aim to ensure that by the end of year 6, as many children as possibly will understand, and use successfully, compact written methods to carry out and record calculations they cannot do in their head.

General Principles

It is important that children approach any calculation by always asking themselves the following questions:

- 'Can I do this in my head?'
- 'Do I know the approximate size of the answer?'
- 'If I can't do it wholly in my head, what do I need to write down in order to help me calculate the answer?'
- 'Will the written method I know be helpful?'
- Whenever appropriate, children should do a mental calculation.
- E.g. Although the numbers in the following calculation are large it is nonetheless easy to calculate mentally: 3008 – 2992
- In order to support this approach, calculations should, as often as possible, be presented to children **horizontally** so that they can make decisions about how to tackle them.

- Estimation should always precede calculation. Children will be encouraged to develop their mental methods and estimation skills throughout the maths curriculum to enable them to check the reasonableness of their answers to calculations.
- To enable children to move towards compact written methods with full understanding, a step-by-step approach is taken. For each of the four operations (+, -, x and ÷) children are first introduced to expanded methods (as these link closely with mental methods) that lead to the compact form of calculation. It is important that children feel secure and comfortable with each stage towards compact methods before they move on to the next.

It is important to remember that children will progress through the stages of expanded calculation at different rates.

It is vital that they can operate efficiently at any stage and with understanding, than to move on too quickly.

- Expanded methods for addition and subtraction are introduced when children are ready. Prior to this children will be doing informal recording of their mental calculations. Expanded methods for multiplication and division are also introduced as soon as children move on to working beyond the multiplication facts up to 12x12 (which are now expected to be learnt by the end of Year 4)
- It is important that as children learn the steps of compact methods, they do not lose the sense of the size of the numbers involved.
- Before compact written methods are taught, it is vital that the children have the necessary mathematical understanding in place.
- Children will be given access to a range of calculation methods (as outlined in this policy).

Written Methods for Addition and Subtraction

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition and subtraction which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition and subtraction of whole numbers which for most children will be by the end of Year 4. The method can then be extended to include decimal numbers (for most children in Year 5 and Year 6).

Criteria which would indicate a child's readiness for formal written methods of addition and subtraction include:

- Ability to add at least three single-digit numbers mentally
- Ability to add and subtract any pair of two-digit numbers mentally
- Knowledge of addition and subtraction facts to 20
- Understanding of place value and ability to partition numbers into hundreds, tens and ones in different ways (very important for adjustments in subtraction
    - eg partition 74 into 70+4 or 60+14)
- Ability to add or subtract multiples of 10 (such as 60 and 70) or of 100 (such as 600 and 700) using the related addition fact, 6 + 7, and their knowledge of place value
- Understanding of commutative and associative laws of addition.
- Ability to explain mental strategies orally and in writing
- Ability to estimate answers to calculations confidently using rounding up or rounding down.
- Ability to check answers to calculations with a reasonable degree of accuracy

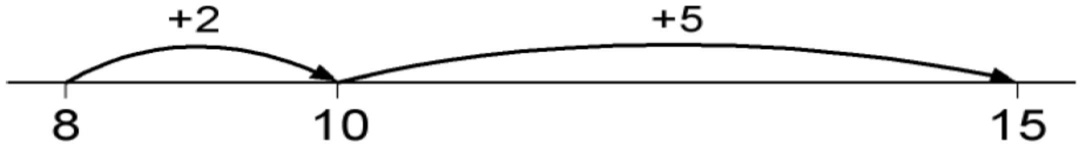
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition and subtraction.

<u>Progression in addition</u>

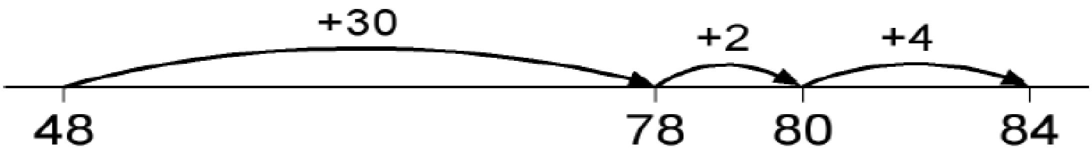
Stage 1 – The empty number line

The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.

8 + 7 = 15



48 + 36 = 84



or:

Stage 2 - Partitioning

The next stage is to record mental methods using partitioning. Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.

Record steps in addition using partitioning:

47 + 76 = 40 + 70 + 7 + 6 = 123

47 + 76 = 47 + 70 + 6
= 117 + 6 = 123

47 + 76 → 40 + 70 = 110
7 + 6 = 13
123

Partitioned numbers are then written under one another:

```
   47   =   40 +  7
 + 76       70 +  6
           110 + 13  = 123
```

Stage 3 – Expanded methods in columns

- Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. To find the partial sums, either the tens or the ones can be added first.
- The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'.
- In the early stages, children should be encouraged to record the partial sums in brackets as this reinforces their understanding of the place value involved.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Adding the tens first:

```
   47
 + 76
  110   (40 + 70)
   13    (7 + 6)
  123
```

Adding the units (ones) first:

```
   47
 + 76
   13   (7 + 6)
  110   (40 + 70)
  123
```

Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to consistently adding the ones digits first.

While teaching the column method, ensure children understand the importance of the '=' line. It helps to show the steps in the calculation and avoids errors when calculating each total.

The method can be extended to adding money (however children must be confident at partitioning money amounts into £ and p) :

£1.47 + £2.84

```
   £1.47
+  £2.84
   -----
    0.11  (7p + 4p)
   £1.20  (40p + 80p)
   £3.00  (£1 + £2)
   -----
   £4.31
```

Stage 4 – Column Method

- In this method, recording is reduced further.
- Carry digits are recorded above the calculation, using the words 'carry ten' or 'carry one hundred', not 'carry one' (Children must know the place value of each digit at all times).
- Later, extend to adding three two-digit numbers, two three-digit numbers and then 1,000s, 10,000s and 100,000s.
- Extend to adding decimals to 2 digit places, remembering that the words used should reflect the place value of each digit e.g. tenths and hundredths

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

```
  11          11          11
  47         258         366
+ 76        + 87       + 458
----        ----       -----
 123         345         824
```

Adding decimals :

```
  11             1 1
  23.45         £1.47
+ 17.38       + £2.94
  -----         -----
  40.83         £4.41
```

When adding decimals, continue to refer to the correct place value terminology, for example 5 hundredths add 8 hundredths not 5 add 8. Understanding can be reinforced by using the expanded column method in Stage 3.

<u>Progression in subtraction</u>

Stage 1: The empty number line – counting back and counting-up

- The empty number line is a very useful tool when recording steps in subtraction and finding the difference between quantities in a range of contexts. From KS1 onwards it is important to teach children how to count back and how to count on so that they are able to make a decision about which is the more efficient method to derive a solution.
- A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.
- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.

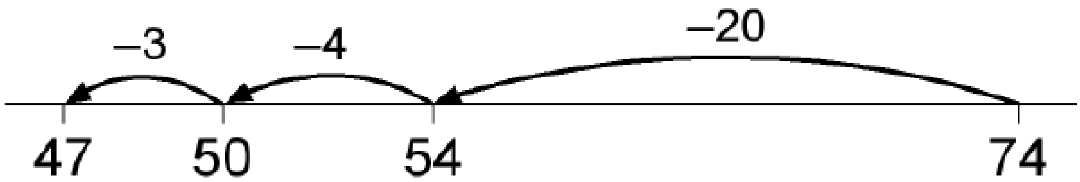
Stage 1: The empty number line – counting back

Steps in subtraction can be recorded on a number line.

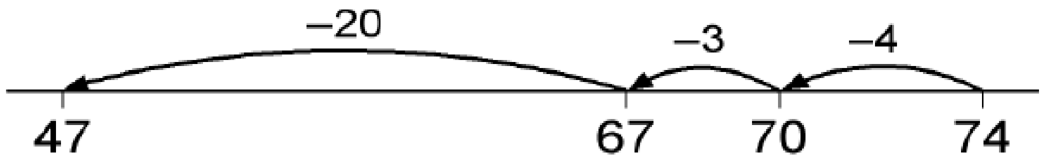
<u>15 – 7 = 8</u>

The steps often bridge through a multiple of 10.

74 – 27 = 47 worked by counting back



The steps may be recorded in a different order:

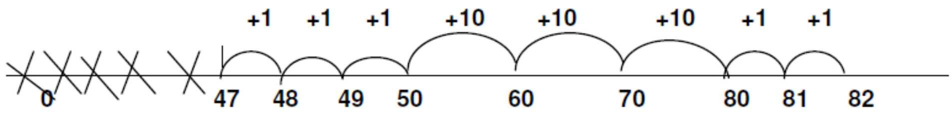


Stage 1: The counting-up method (difference)

The mental method of finding the difference by counting up from the smaller to the larger number can be recorded as follows:

Example 82 - 47

Count up from 47 to 82 in jumps of 10 and jumps of 1.
The number line should still show 0 so children can cross out the section from 0 to the smallest number.
They then associate this method with 'taking away'.

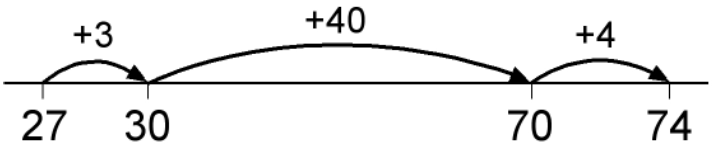


1+1+1+10+10+10+1+1 = 35

With practice, children can reduce the amount of recording and show the steps counting up from the smaller number to the larger number.
The number of jumps can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + _ = 74 mentally.

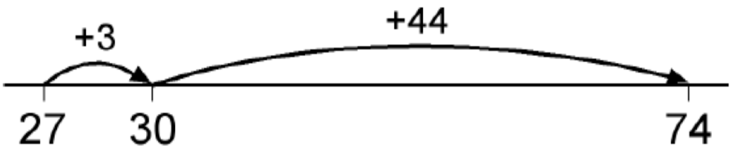
Examples:

2 digit numbers : 74 - 27



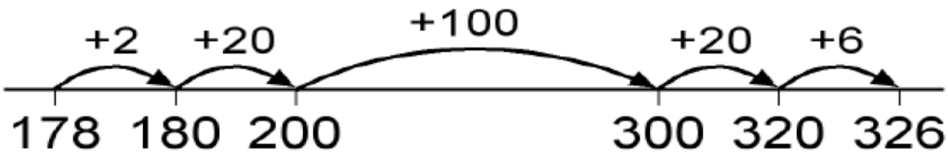
4 + 40 + 3 = 47

Or



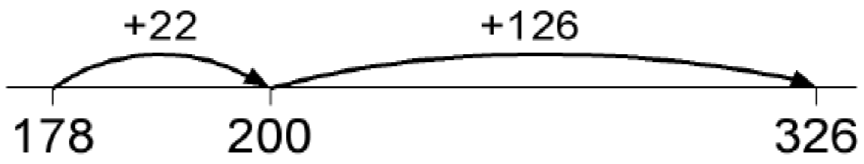
3 + 44 = 47

3 digit numbers : 326 – 178



2 + 20 + 100 + 20 + 6 = 146

Where children have sound knowledge of pairs that total 100, the number of jumps can be reduced further.



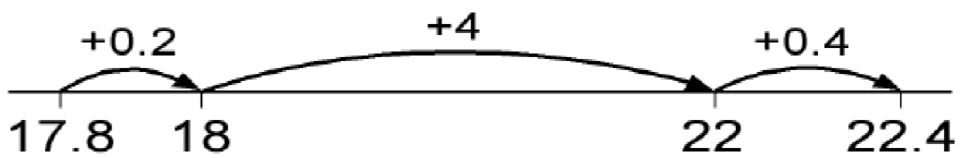
22 + 126 = 148

Stage 1: The counting-up method - additional uses of a number line

Decimal Numbers

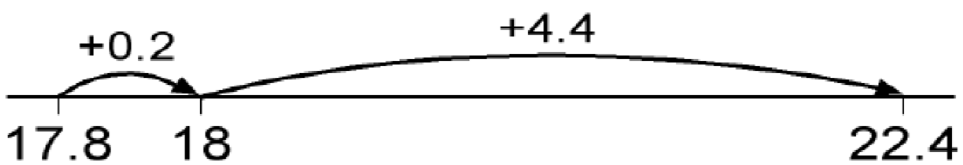
- The method can be used with decimals where no more than three columns of place value are required. However, it becomes less efficient when more than three columns are needed.
- This counting-up method can be a useful alternative for children whose progress is slow and whose mental and written calculation skills are weak.

22.4 – 17.8 =



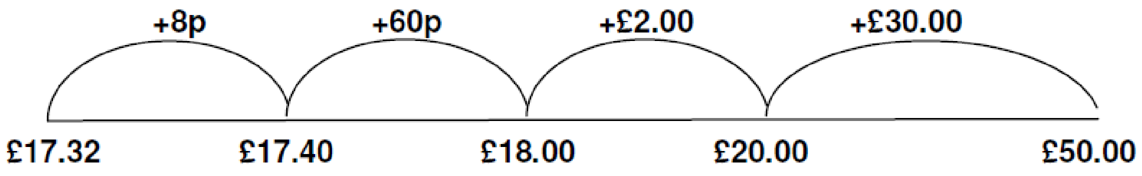
0.2 + 4 + 0.4 = 4.6

Or



0.2 + 4.4 = 4.6

Finding change in money problems

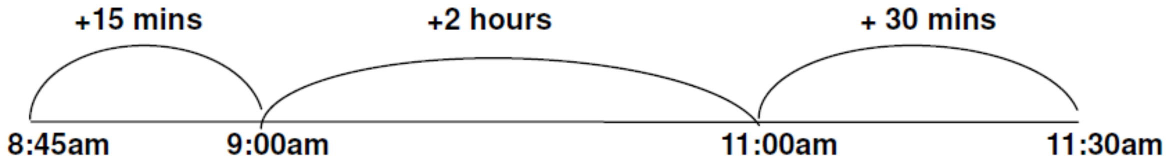


Change = £30.00 + £2.00 + 60p + 8p

= £32.68

<u>Calculating Time Intervals</u>

8:45am to 11:30am



Time Interval = 2 hours + 30 mins + 15 mins

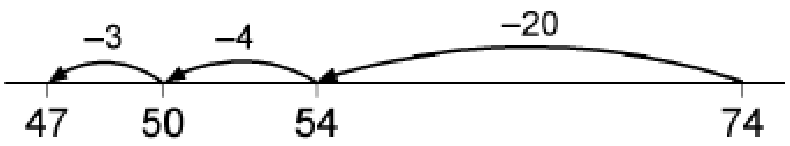
= 2 hours 45 mins

Stage 2: Partitioning

Subtraction can be recorded using partitioning by writing equivalent calculations that can be carried out mentally. For example, 74 – 27: this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn.

74 – 27 = 74 – 20 – 7 = 54 – 7 = 47
or = 54 - 4 - 3 = 47

This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. This method of recording links to counting back on the number line:



NB. Never split the first number. Eg. 93 — 76 = 93 - 70 - 6 = 23 — 6 = 17

When children are secure in their understanding, point out that the partitioned number could be subtracted in any order (e.g. units first) – this will provide a link to the following column methods.

Stage 3: Partitioned (Expanded) layout leading to column method

- Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.
- This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.
- Always refer to the "top number take away the bottom number" eg "50 take away 30", not "take 30 from 50"
- Begin subtraction in the units column. Refer to tens and hundreds eg "300 take away 100" not "3 take away 1"

Two-digit numbers with and without adjustment

Example: 74 – 22 "74 take away 22"

Expanded method leading to

70 + 4	7 4	"4 take away 2"
- 20 + 2	- 2 2	"70 take away 20"
50 + 2 = 52	5 2	(7 tens take away 2 tens)

Example: 74 – 27

	60 14	**6 14**
7 0 + 4	~~7 0~~ + ~~4~~	7 4
2 0 + 7	- 2 0 + 7	- 2 7
	4 0 + 7	4 7

Ensure that children are clear that the adjusted number (60+14) is the same amount as the original number (70+4)

Three-digit numbers, no adjustment needed

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take away forty', making the link to the related fact 6 take away 4.

Example: 563 - 241	Expanded method	leading to
	500 + 60 + 3	563
	- 200 + 40 + 1	- 241
	300 + 20 + 2	322

Three-digit numbers, adjustment needed

Explain that children are using their skills of re-partitioning to help with the subtraction. The re-partitioned number is the same value (700+40+1 = 600+140+1).

Example 563 – 271

Expanded method

500 + 60 + 3
\- 200 + 70 + 1

400 160
~~500~~ + ~~60~~ + 3
\- 200 + 70 + 1
200 + 90 + 2 = 292

leading to

4 16
~~5~~ ~~6~~ 3
\- 2 7 1
2 9 2

Example 741 – 367

700 + 40 + 1
− 300 + 60 + 7

600 130 11
~~700~~ + ~~40~~ + ~~1~~
− 300 + 60 + 7
300 + 70 + 4

6 13 11
~~7~~ ~~4~~ ~~1~~
− 3 6 7
3 7 4

Example 741 – 367

To explain zero as a middle number on top line:

500 + 00 + 6
\- 100 + 20 + 9

90 16
400 ~~100~~ ~~6~~
~~500~~ + ~~00~~ + 6
\- 100 + 20 + 9
300 + 70 + 7

4 9 16
~~5~~ 1~~0~~ ~~6~~
\- 1 2 9
3 7 7

In this case, zero acts as a place holder for the tens. The adjustment has to be done in two stages.

First the 500 + 00 is partitioned into 400 + 100 and then the 100 + 6 is partitioned into 90 + 16.

The column method can be extended to the subtraction of decimal numbers. Ensure children are clear that the Stage 1 Counting up method on the Number Line is more efficient and accurate when finding the difference between numbers that are close together or near to multiples of 10, 100 etc (eg 4003 - 2995, or £10.00 – £7.68).

Written Methods for Multiplication

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 12 × 12;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their
- knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

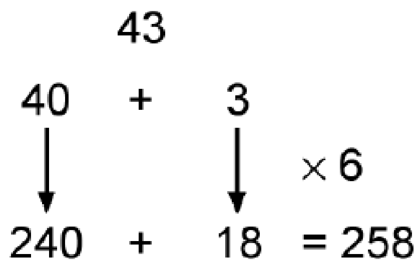
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Stage 1: Mental multiplication using partitioning (TU x U)

Children need to learn and practise their times tables very regularly. Use arrays and visual models to explain how related facts can be derived eg from 3x5 = 15, we know 3x50=150, 30x5=150,

The tens and ones can be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.

Informal recording in Year 3/4 might be:



Also record mental multiplication using partitioning:

14 x 3 = (10 + 4) x 3
= (10 x 3) + (4 x 3) = 30 + 12 = 42

43 x 6 = (40 + 3) x 6
= (40 x 6) +(3 x 6) = 240 + 18 = 258

Stage 2: The grid method (TU x U)

- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.

- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

×	7
30	210
8	56
	266

Stage 3: Expanded short multiplication (TU x U)

- The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
- Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.
- Most children should be able to use this expanded method for TU × U by the end of Year 4.

Expanded method		Leading to	
30 + 8		38	
X 7		x 7	
56	(7 x 8)	56	(7 x 8)
210	(7 x 30)	210	(7 x 30)
266		266	

Stage 4: Short multiplication (TU x U)

- The recording is reduced further, with carry digits recorded below the line.
- If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.

```
  38
× _7
 266
  5
```

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.

Stage 5: Grid Method (TU x TU)

- The grid method requires each multiplier to be partitioned into tens, units etc to enable accurate multiplying, clearly showing the effect of place value when multiplying.
- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added and then the two sums at the end of each row are added to find the total product.

56 × 27 is approximately 60 × 30 = 1800.

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

- The grid method should not be used to solve HTU × TU problems as there is an increased chance of errors due to the long process of adding partial products.

Stage 6: Expanded Method (TU x TU, HTU x TU)

Example TU x TU — 56 × 27 is approximately 60 × 30 = 1800.

- Reduce the recording, showing the links to the grid method above.
- When recording in columns, units should line up under units, tens under tens etc.
- Children should record the partial product calculations to reinforce and demonstrate their understanding of using place value to derive multiplications from known facts (eg 5x7 = 35, so 50x7 = 350)
- The final step requires the carry digits in the partial products of 7x56=392 and 20x56=1120 to be carried mentally.

Begin with (showing links to grid method):

```
  56
x 27
  42   (7 x 6)
 350   (7 x 50)
 120   (20 x 6)
1000   (20 x 50)
1512
 1
```

Leading to:

```
   56
x  27
  392    ( 7 x 56)
   4
 1120    (20 x 56)
  1
 ____
 1512
 1
```

Leading to the final step:

```
  56
x 27
 392   ( 7 x 56)
1120   (20 x 56)
1512
 1
```

Example HTU x TU — 286 × 29 is approximately 300 × 30 = 9000.

- Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.
- Again, the carry digits in the partial products are usually carried mentally in the final step.

Showing carry digits:

```
  286
X  29
 2574   ( 9 x 286)
  7 5
 5720   (20 x 286)
 1 1
 ____
 8294
 1
```

Carry digits carried mentally:

```
  286
x  29
 2574   ( 9 x 286)
 5720   (20 x 286)
 8294
 1
```

Stage 7: Formal Written Method (TU x TU, HTU x TU) - long multiplication

- The recoding is reduced further, with carrying digits below or above.
- If after attempting this method the children are cannot use this compact method without making errors, they should return to the expanded format seen on stage 6)

```
   3
   5
   38
 x 47
 ----
  266
 1520
 ----
 1786
```

Written Methods for Division

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

The following content show the stages in building up to long division through Years 3 to 6 – first mental or number line methods, then long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and finally short division HTU ÷ U.

Sharing and Grouping

Children need to be taught that division can be considered as 'sharing' and 'grouping' (how many groups of the divisor make up the dividend. eg 18 ÷ 3 means 'how many 3s are in 18' and 18 shared between 3) The concept of grouping is very important when carrying out written methods of division and should certainly be taught and discussed in KS1.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and
- divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

Stage 1: Mental division using partitioning

One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining ones, then divide each part separately.

Informal recording in Year 4 for 84 ÷ 7 might be:

84

70 + 14

↓ ↓ ÷ 7

10 + 2 = 12

In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.

This is a useful method for halving any number:

376 ÷ 2

300 + 70 + 6

↓ ↓ ↓ ÷ 2

150 + 35 + 3 = 188

An alternative method for informal recording of division by partitioning might be:

64 ÷ 4 = (40 + 24) ÷ 4
= (40 ÷ 4) + (24 ÷ 4)
= 10 + 6 = 16

87 ÷ 3 = (60 + 27) ÷ 3
= (60 ÷ 3) + (27 ÷ 3)
= 20 + 9 = 29

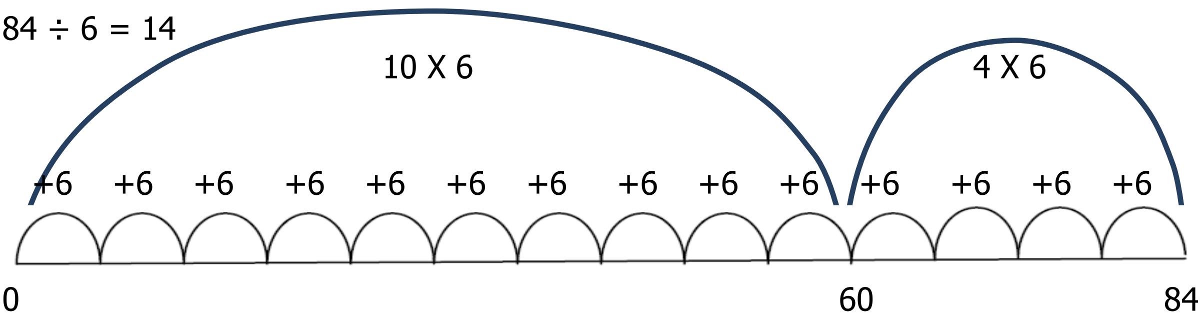
Stage 1: Mental division by grouping/chunking on a number line

TU ÷ U on a number line

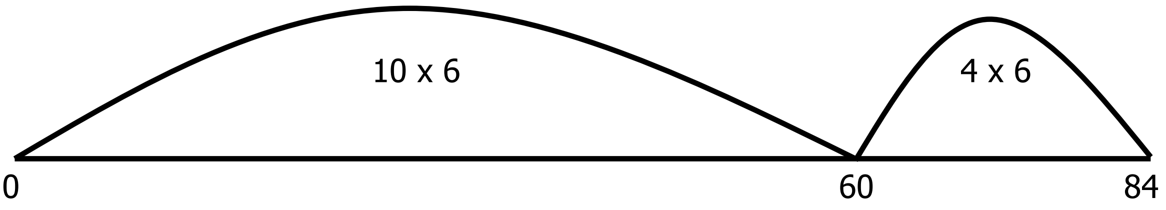
This method uses the grouping model; children count up in multiples of the divisor until the dividend is reached.

The number line then shows how many groups of the divisor there are making up the dividend.

84 ÷ 6 = 14

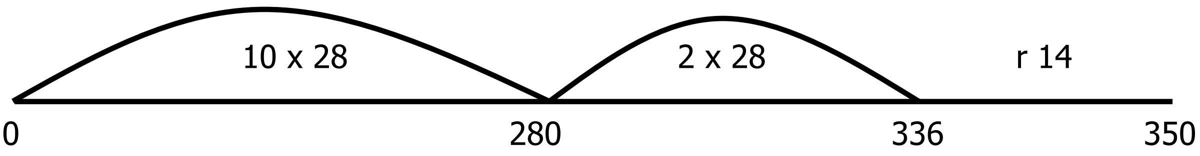


With confidence, the recording can be reduced further:



HTU ÷ TU on a number line

350 ÷ 28 = 12 r 14



When children are secure, point out and demonstrate that the same result can be achieved by repeatedly subtracting multiples of the divisor from the dividend. This will provide a link to the following written methods.

Stage 2: Chunking; 'Expanded' method for TU ÷ U and HTU ÷ U

- This method is based on subtracting multiples of the divisor from the number to be divided, the dividend. The answer is then derived by totalling how many groups of the divisor were subtracted from the dividend.
- The method has a clear link to the number line method shown above.
- Use concrete examples to show how division can be represented as repeated subtraction – repeatedly 'take away' groups of the divisor. For example 97 ÷ 9 can be solved by taking away 10 groups of 9 until there is a remainder of 7.
- With confidence, children will be able to recognise that they can subtract multiples of the divisor
- Encourage them to reduce the number of steps. With practice they should look for the biggest multiples of the divisor that they subtract.
- For HTU dividends, it is particularly important to estimate an answer by multiplying the divisor by multiples of 10 to find the 2 multiples that 'trap' the dividend.

Example TU ÷ U 97 ÷ 9 Estimate : 10 (90÷9) or 11 (99÷9)

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9)97
 - 90 (10 x 9)
   7
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Answer: 10 r 7

Example HTU ÷ U 196 ÷ 6

Estimate : 196 is 'trapped' by 180 and 240 so the estimate is between 30 (180÷6) and 40 (240÷6)

Begin with	**Leading to**
To find 196 ÷ 6, we subtract multiples of 6, 6)196 - 60 (10 x 6) 136 - 60 (10 x 6) 76 - 60 (10 x 6) 16 - 12 (2 x 6) 4 Answer: 32 r 4	Simplify the division by subtracting the largest possible multiple of 6, which is 180, leaving 16. Then the largest possible multiple of 6 is now 12, leaving a remainder of 4. 6)196 - 180 (30 x 6) 16 - 12 (2 x 6) 4 Answer: 32 r 4

The chunking method can be successfully extended to larger calculations (eg HTU ÷ TU) and provides a direct link to informal (number line) methods of recording. Progression onto the formal short and long division methods can be introduced to children who are confident with multiplication and division facts, and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

Stage 3 : Short division of TU ÷ U

This method links to mental methods using partitioning.

Example 81 ÷ 3 **Estimate:** between 20 (60 ÷ 3) and 30 (90 ÷ 3)

81 is partitioned into 60 (the highest multiple of 3 that is also a multiple of 10 and less than 81), to give 60 + 21

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  20 + 7
3)60 + 21
```

The recording is then shortened to

```
   2 7
3) 8 ²1
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Teacher: 'How many threes divide into 80 so that the answer is a multiple of 10?'

This gives 20 threes or 60, with 20 remaining.

We now ask: 'What is 21 divided by three?' which gives the answer 7.

Stage 4: Short division of HTU ÷ U
Example 291 ÷ 3 **Estimate**: between 90 (270 ÷ 3) and 100 (300 ÷ 3) 291 is partitioned into 270 (the highest multiple of 3 that is also a multiple of 10 and less than 291), to give 270 + 21 $\begin{array}{r} 90 + 7 \\ 3\overline{)270 + 21} \end{array}$ The recording is then shortened to $\begin{array}{r} 9\ 7 \\ 3\overline{)2\ 9\ {}^{2}1} \end{array}$ We ask: 'How many threes in 290? (The answer must be a multiple of 10)' This gives 90 threes or 270, with 20 remaining. We now ask: 'What is 21 divided by three?' which gives the answer 7.
Stage 5: Long division of HTU ÷ TU
• The next step is to tackle HTU ÷ TU, which for most children will be in Year 6. • The first layout below the 'chunking' method for 'long division'. • Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording. Example: How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. 24)560 -480 (20 x 24) 80 - 72 (3 x 24) 8 Answer: 23 r 8 In effect, the recording above is the 'chunking method' for long division, though conventionally the digits of the answer are recorded above the line as shown below. 23 24)560 -480 80 - 72 8 Answer: 23 r 8

Stage 5: Long division of HTU ÷ TU (cont.)

Below is the recording using the 'formal' method for long division:

432 ÷ 15 = 28·8

```
         2 8 · 8
      ___________
15  )   4 3 2 · 0
        3 0
        1 3 2
        1 2 0
          1 2 0
          1 2 0
              0
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NB Only teach this method when children are completely secure with the previous method.

The remainder is expressed as a decimal.

Children will need to select the most effective method for each calculation/problem they meet, including whether to use the standard, formal written method of long division.

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